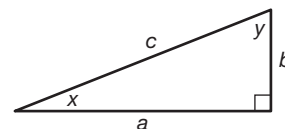Name _____   Date _____

Three Angle Measure
## Introduction to Trigonometry

Vocabulary

Use the diagram to complete each sentence.

1. If *b* is the opposite side, then *x* is the _____ .
2. If *y* is the reference angle, then *b* is the _____ .
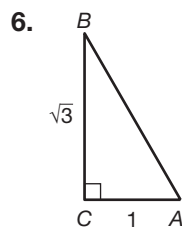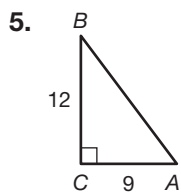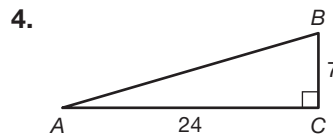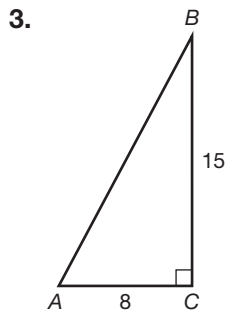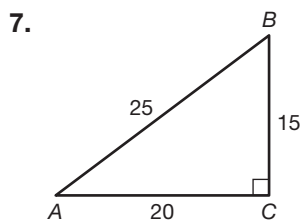3. If *x* is the reference angle, then *b* is the _____ .

Problem Set

Determine the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}}$ using $\angle A$ as the reference angle in each triangle. Write your answers as fractions in simplest form.
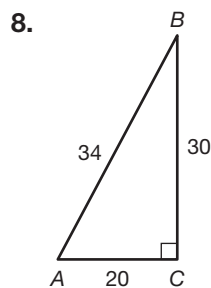
1.

$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

2.

3.

4.

5.

6.
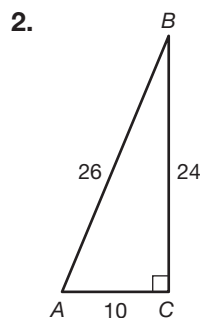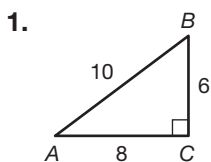
Determine the ratio $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ using $\angle A$ as the reference angle in each triangle. Write your answers as fractions in simplest form.
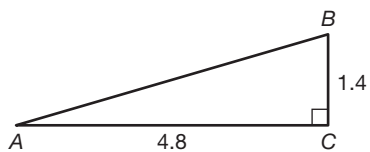
7.

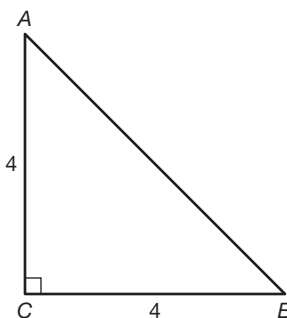$$\dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{20}{25} = \dfrac{4}{5}$$

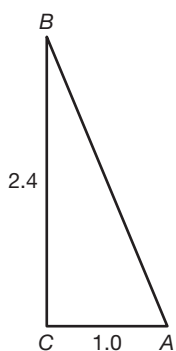**8.**
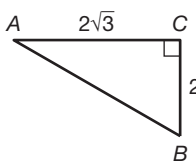
Name _____ Date _____

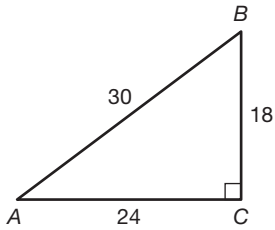**9.**

10.

11.

12.

Determine the ratios $\dfrac{\text{opposite}}{\text{hypotenuse}}$, $\dfrac{\text{adjacent}}{\text{hypotenuse}}$, and $\dfrac{\text{opposite}}{\text{adjacent}}$ using $\angle A$ as the reference angle in each triangle. Write your answers as fractions in simplest form.
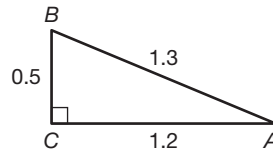
13.

$$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{18}{30} = \frac{3}{5}$$
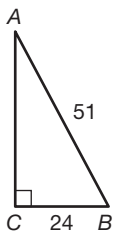
$$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{24}{30} = \frac{4}{5}$$

$$\frac{\text{opposite}}{\text{adjacent}} = \frac{18}{24} = \frac{3}{4}$$
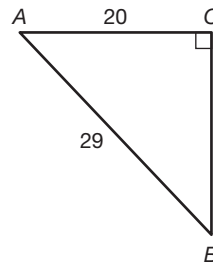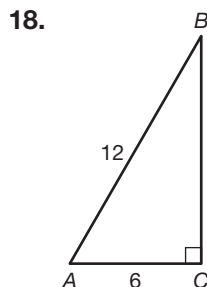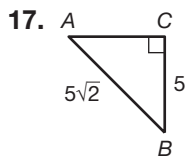
14.

15.

16.

Name _____ Date _____

17.

18.

In each figure, triangles *ABC* and *DEF* are similar by the *AA* Similarity Theorem. Calculate the indicated ratio twice, first using △*ABC* and then using △*ADE*.

19. $\dfrac{\text{opposite}}{\text{hypotenuse}}$ for reference angle *A*
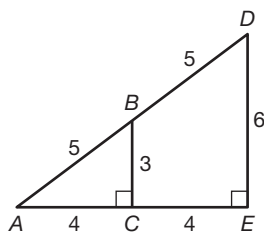
$AE = 4 + 4 = 8$

$AD = 5 + 5 = 10$

In △*ABC*, $\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{5}$.

In △*ADE*, $\dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{6}{10} = \dfrac{3}{5}$.

20. $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ for reference angle *A*

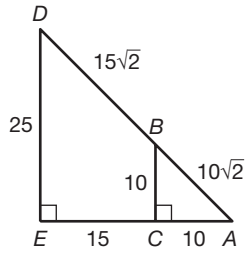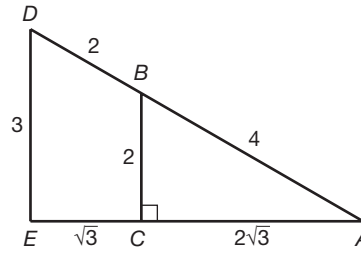**21.** $\dfrac{\text{opposite}}{\text{hypotenuse}}$ for reference angle *A*

D

15√2

25   *B*

10   10√2
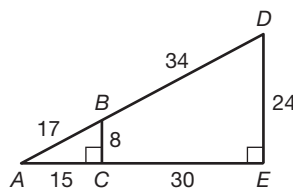
E   15   *C*   10   *A*

22. $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ for reference angle *A*

D

2

B

3   4

2

E   √3   *C*   2√3   *A*

23. $\dfrac{\text{opposite}}{\text{adjacent}}$ for reference angle *A*

D

34

B   24

17   8

A   15   *C*   30   *E*
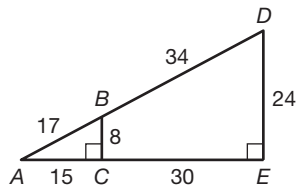
24. $\dfrac{\text{opposite}}{\text{adjacent}}$ for reference angle *A*

D

8.7   8.4

B

2.9   2.1

A   2.0   *C*   6.0   *E*
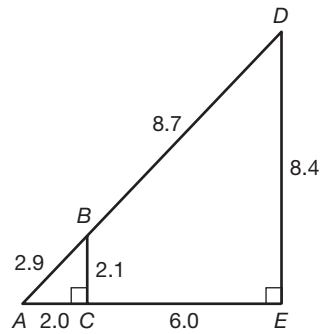
Name _____ Date _____

The Tangent Ratio
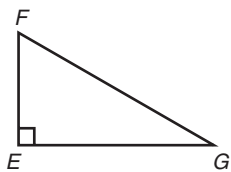## Tangent Ratio, Cotangent Ratio, and Inverse Tangent

Vocabulary

Match each description to its corresponding term for triangle *EFG*.
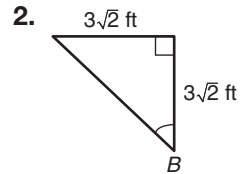

1. $\dfrac{EG}{EF}$ in relation to $\angle G$

a. tangent

2. $\dfrac{EF}{EG}$ in relation to $\angle G$

b. cotangent

3. $\tan^{-1}\left(\dfrac{EF}{EG}\right)$ in relation to $\angle G$

c. inverse tangent

Problem Set

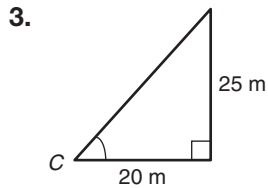Calculate the tangent of the indicated angle in each triangle. Write your answers in simplest form.
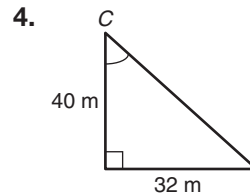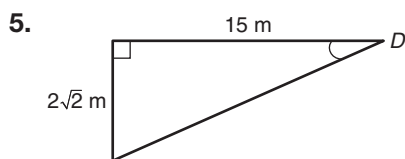
1.

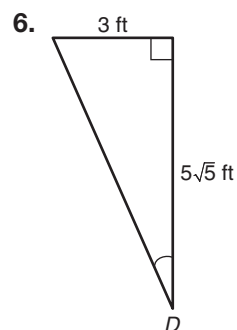$\tan B = \dfrac{2}{2} = 1$

2.

$\tan B =$

3.

$\tan C =$
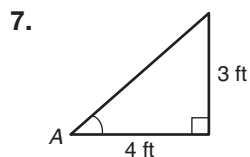
4.

$\tan C =$

5.

$\tan D =$

6.
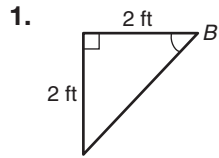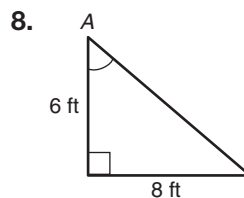

$\tan D =$

Name _____ Date _____

Calculate the cotangent of the indicated angle in each triangle. Write your answers in simplest form.

7.

3 ft, 4 ft, A

$\cot A = \dfrac{4}{3}$

8. A

6 ft, 8 ft

$\cot A =$

9. F

7 yd, 15 yd

$\cot F =$

10.

6 yd, $2\sqrt{6}$ yd, F

$\cot F =$

11. $4\sqrt{2}$ ft, $4\sqrt{2}$ ft, A

$\cot A =$

12. 32 m, A, 40 m
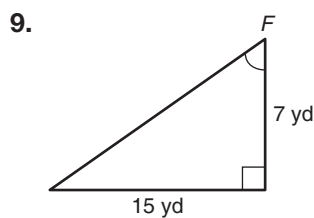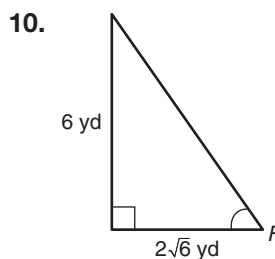
$\cot A =$

Use a calculator to approximate each tangent ratio. Round your answers to the nearest hundredth.

13. tan 30°
0.58

14. tan 45°

15. tan 60°

16. tan 15°

17. tan 75°

18. tan 89°

Use a calculator to approximate each cotangent ratio. Round your answers to the nearest hundredth.
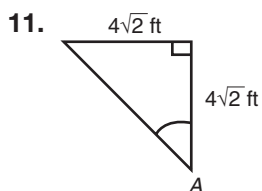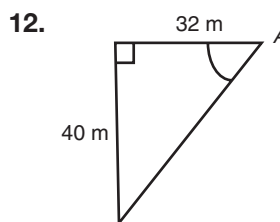
19. cot 60°

0.58

20. cot 15°

21. cot 45°

22. cot 75°

23. cot 10°

24. cot 30°

Use a tangent ratio or a cotangent ratio to calculate the missing length of each triangle. Round your answers to the nearest hundredth.

25.

$\tan 40° = \dfrac{x}{2}$

$2 \tan 40° = x$

$x \approx 1.68$ ft

26.

27.

28.

Name _____ Date _____

29.

x
$\sqrt{11}$ yd
$25°$

30.

x
$63°$
$3\sqrt{2}$ yd

Calculate the measure of angle *X* for each triangle. Round your answers to the nearest hundredth.

31.
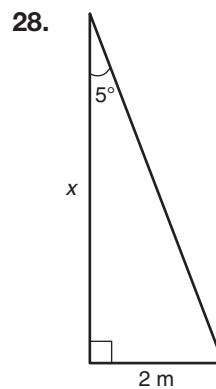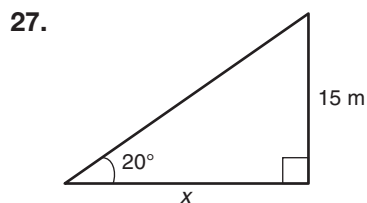

B
9 in.
5 in.
X
T

$\tan X = \dfrac{5}{9}$

$m\angle X = \tan^{-1}\left(\dfrac{5}{9}\right) \approx 29.05°$

32.

X
30 m
S
43 m
R

33.

K
$8\sqrt{3}$ cm
M
$6\sqrt{2}$ cm
X

34.

Y
5.94 km
Z
5.66 km
X

35.

36.
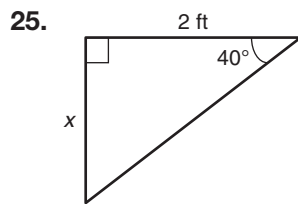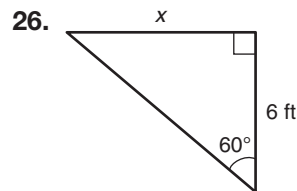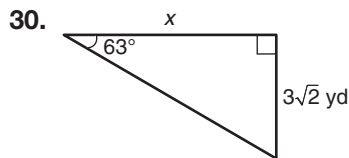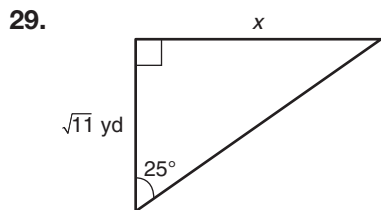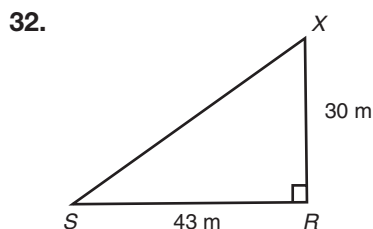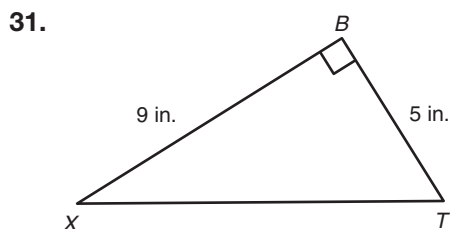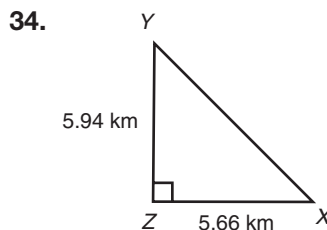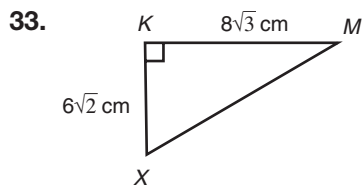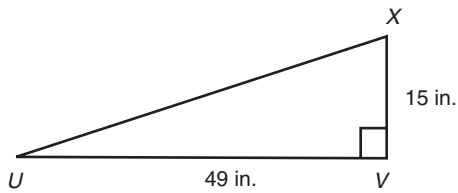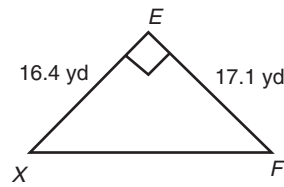
Solve each problem. Round your answers to the nearest hundredth.

37. A boat travels in the following path. How far north did it travel?
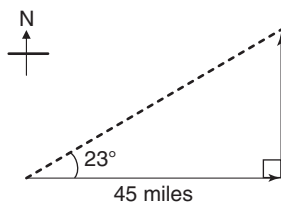
$$\tan 23° = \frac{N}{45}$$

$$45 \tan 23° = N$$

$$N \approx 19.10 \text{ mi}$$

38. During a group hike, a park ranger makes the following path. How far west did they travel?
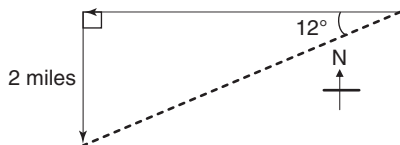
39. A surveyor makes the following diagram of a hill. What is the height of the hill?
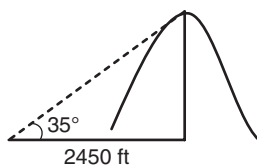
Name _____ Date _____

40. To determine the height of a tree, a botanist makes the following diagram. What is the height of the tree?

70°
20 ft
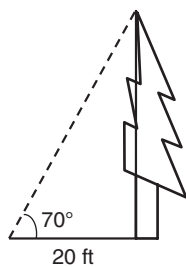
41. A moving truck is equipped with a ramp that extends from the back of the truck to the ground. When the ramp is fully extended, it touches the ground 12 feet from the back of the truck. The height of the ramp is 2.5 feet. Calculate the measure of the angle formed by the ramp and the ground.

2.5 ft

?

12 ft
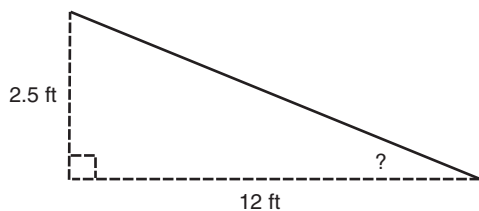
42. A park has a skateboard ramp with a length of 14.2 feet and a length along the ground of 12.9 feet. The height is 5.9 feet. Calculate the measure of the angle formed by the ramp and the ground.

14.2 ft

5.9 ft

?

12.9 ft
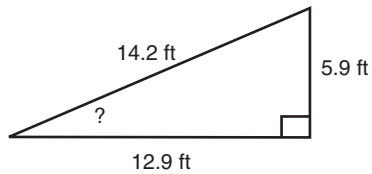
43. A lifeguard is sitting on an observation chair at a pool. The lifeguard's eye level is 6.2 feet from the ground. The chair is 15.4 feet from a swimmer. Calculate the measure of the angle formed when the lifeguard looks down at the swimmer.

?

6.2 ft

15.4 ft

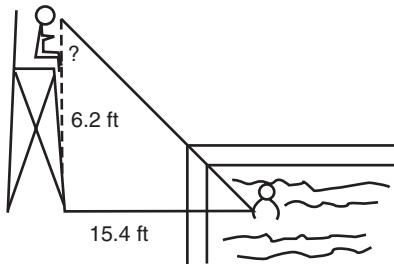**44.** A surveyor is looking up at the top of a building that is 140 meters tall. His eye level is 1.4 meters above the ground, and he is standing 190 meters from the building. Calculate the measure of the angle from his eyes to the top of the building.

140 m

?

190 m

1.4 m
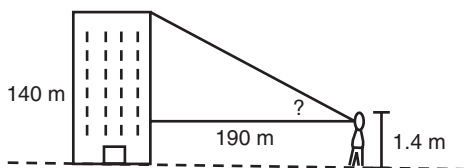
Name _____ Date _____

The Sine Ratio
Sine Ratio, Cosecant Ratio, and Inverse Sine

Vocabulary

Write the term from the box that best completes each statement.

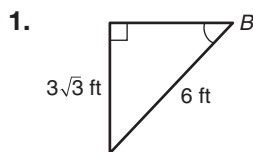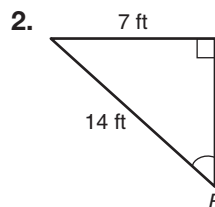| sine | cosecant | inverse sine |

1. The _____ of an acute angle in a right triangle is the ratio of the length of the hypotenuse to the length of a side that is opposite the angle.

2. The _____ of x is the measure of an acute angle whose sine is x.

3. The _____ of an acute angle in a right triangle is the ratio of the length of the side that is opposite the angle to the length of the hypotenuse.
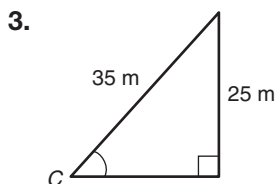
Problem Set

Calculate the sine of the indicated angle in each triangle. Write your answers in simplest form.
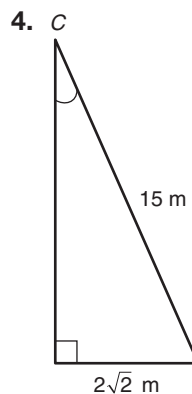
1.

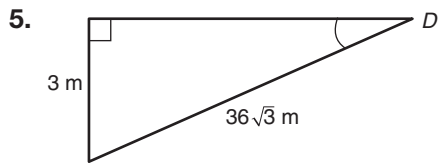$$\sin B = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

2.

$$\sin B =$$

3.

$$\sin C =$$

4.

$$\sin C =$$

5.

3 m

36√3 m

D

sin *D* =

6.

6√3 m

54 m

D
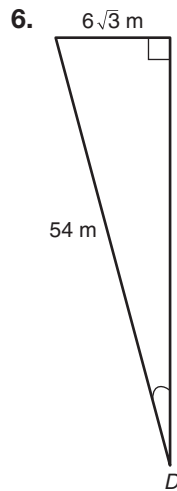
sin *D* =

Calculate the cosecant of the indicated angle in each triangle. Write your answers in simplest form.

7.

12 ft

8 ft

A

$\csc A = \dfrac{12}{8} = \dfrac{3}{2}$

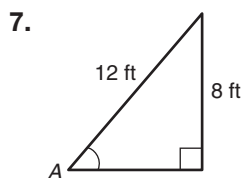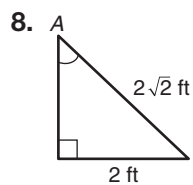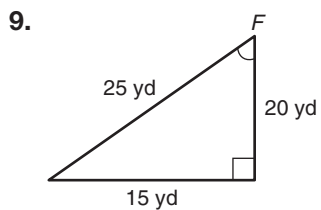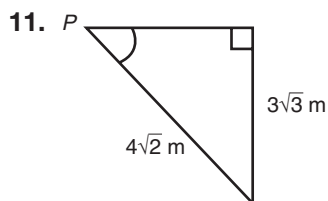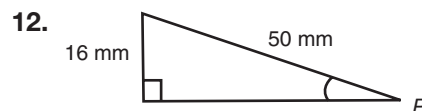**8.** A

2√2 ft

2 ft

$\csc A =$

9. F

25 yd

20 yd

15 yd

$\csc F =$

10.

6√3 yd

12 yd

6 yd

F

$\csc F =$

11. P

3√3 m

4√2 m

$\csc P =$

12.

16 mm

50 mm

P

$\csc P =$

© Carnegie Learning
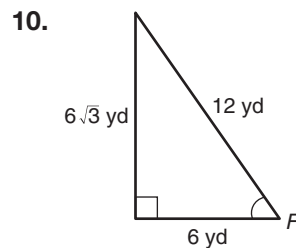
Name _____ Date _____

Use a calculator to approximate each sine ratio. Round your answers to the nearest hundredth.

13. sin 30°

   0.5

14. sin 45°

15. sin 60°

16. sin 15°

17. sin 75°

18. sin 5°

Use a calculator to approximate each cosecant ratio. Round your answers to the nearest hundredth.

19. csc 45°

   1.41

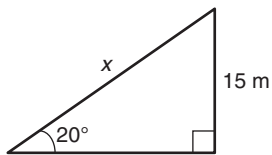**20.** csc 90°

21. csc 120°

22. csc 30°

23. csc 15°

24. csc 60°

Use a sine ratio or a cosecant ratio to calculate the missing length of each triangle. Round your answers to the nearest hundredth.

25.

40°

x

2 ft

$\sin 40° = \dfrac{x}{2}$

$2 \sin 40° = x$

$x \approx 1.29 \text{ ft}$

26.

x

6 ft

60°

27.

15 m

x

20°

28.

5°

x

2 m

29.

$\sqrt{11}$ yd

x

25°

30.

63°

x

$3\sqrt{2}$ m

Calculate the measure of angle X for each triangle. Round your answers to the nearest hundredth.

31.

X 15 ft H

8 ft

R

32.

X

R

42 mm

30 mm

T
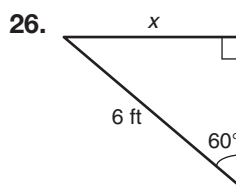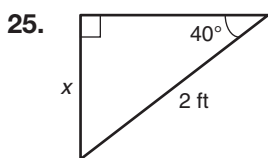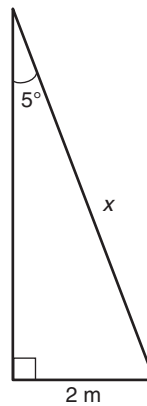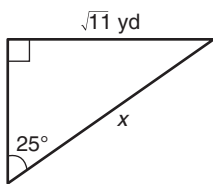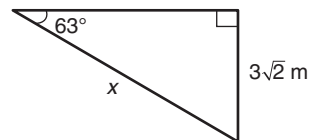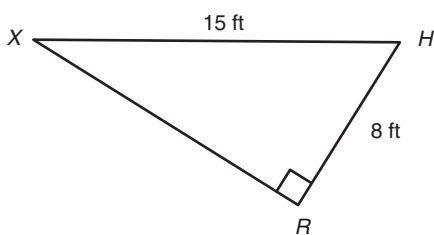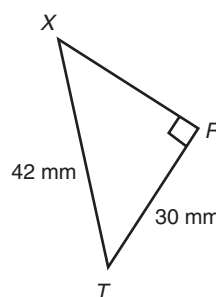
$\sin X = \dfrac{8}{15}$

$m\angle X = \sin^{-1}\left(\dfrac{8}{15}\right) \approx 32.23°$

Name _____ Date _____

33.

34.

35.

36.
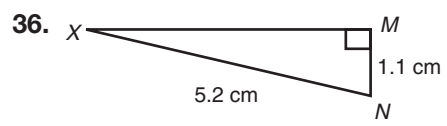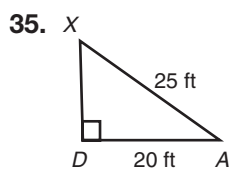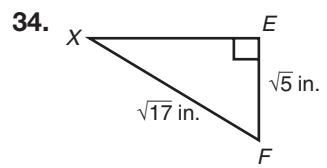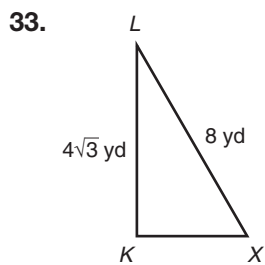
Solve each problem. Round your answers to the nearest hundredth.

37. A scout troop traveled 12 miles from camp, as shown on the map below. How far north did they travel?
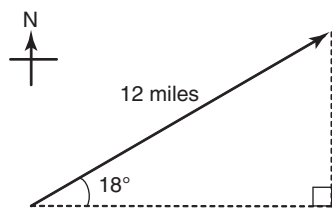
$$\sin 18° = \frac{N}{12}$$

$$12 \sin 18° = N$$

$$N \approx 3.71 \text{ mi}$$

38. An ornithologist tracked a Cooper's hawk that traveled 23 miles. How far east did the bird travel?
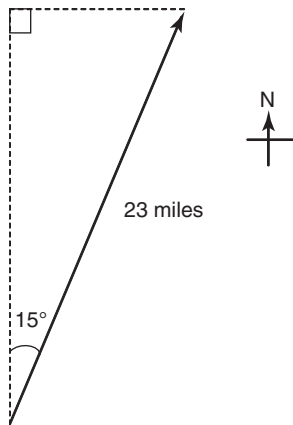
39. An architect needs to use a diagonal support in an arch. Her company drew the following diagram. How long does the diagonal support have to be?
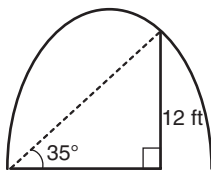
40. A hot air balloon lifts 125 feet into the air. The diagram below shows that the hot air balloon was blown to the side. How long is the piece of rope that connects the balloon to the ground?
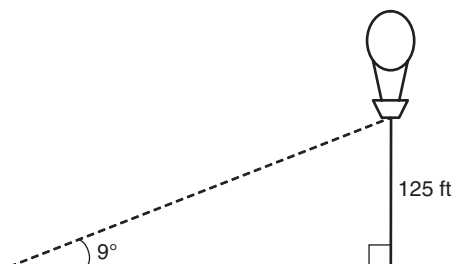
Name _____ Date _____

41. Jerome is flying a kite on the beach. The kite is attached to a 100-foot string and is flying 45 feet above the ground, as shown in the diagram. Calculate the measure of the angle formed by the string and the ground.

45 ft

100 ft

?
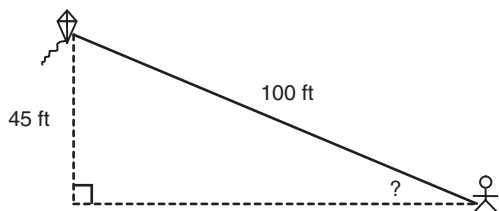
42. An airplane ramp is 58 feet long and reaches the cockpit entrance 19 feet above the ground, as shown in the diagram. Calculate the measure of the angle formed by the ramp and the ground.

58 ft

19 ft

?
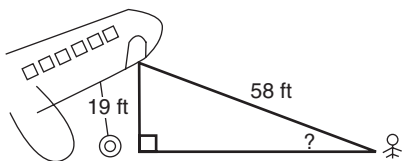
43. Bleachers in a stadium are 4 meters tall and have a length of 12 meters, as shown in the diagram. Calculate the measure of the angle formed by the bleachers and the ground.

4 m

12 m

?
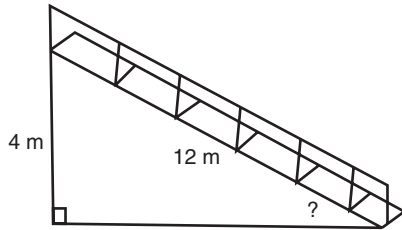
44. A 20-foot flagpole is raised by a 24-foot rope, as shown in the diagram. Calculate the measure of the angle formed by the rope and the ground.

24 ft   20 ft

?
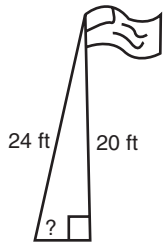
Name _____ Date _____

The Cosine Ratio
Cosine Ratio, Secant Ratio, and Inverse Cosine

Vocabulary

Describe the similarities and differences between the pair of terms.
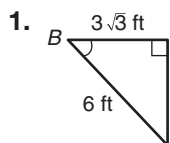
1. cosine ratio and secant ratio

Define the term in your own words.
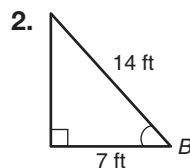
2. inverse cosine

Problem Set

Calculate the cosine of the indicated angle in each triangle. Write your answers in simplest form.

1.

$\cos B = \dfrac{3\sqrt{3}}{6} = \dfrac{\sqrt{3}}{2}$

2.

$\cos B =$

3.

$\cos C =$

4.

$\cos C =$

© Carnegie Learning

5.

cos D =

6.
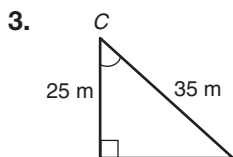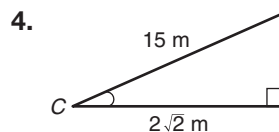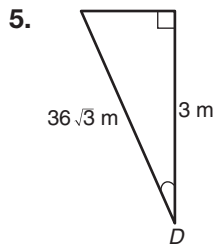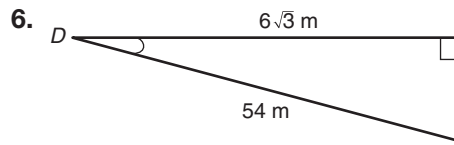
cos D =

Calculate the secant of the indicated angle in each triangle. Write your answers in simplest form.

7.

sec A = $\dfrac{12}{8}$ = $\dfrac{3}{2}$

8.

sec A =
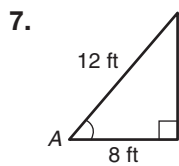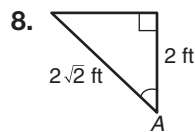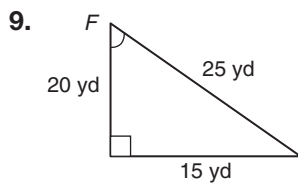
9.

sec F =

10.

sec F =

11.

sec P =

12.
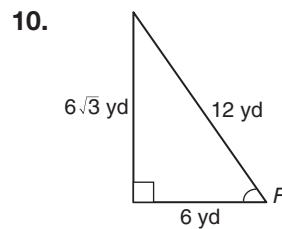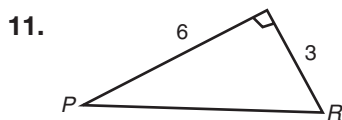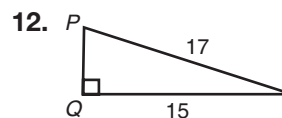
sec P =

Name _____ Date _____

Use a calculator to approximate each cosine ratio. Round your answers to the nearest hundredth.

13. cos 30°

0.87

14. cos 45°

15. cos 60°

16. cos 15°

17. cos 75°

18. cos 89°

Use a calculator to approximate each secant ratio. Round your answers to the nearest hundredth.

19. sec 45°

$\dfrac{1}{\cos(45°)} = 1.41$

20. sec 25°

21. sec 75°

22. sec 30°

23. sec 15°

24. sec 60°

Use a cosine ratio or a secant ratio to calculate the missing length of each triangle. Round your answers to the nearest hundredth.

25.

2 ft

40°

x

$\cos 40° = \dfrac{x}{2}$

$2 \cos 40° = x$

$x \approx 1.53$ ft

26.

6 ft

60°

x

27.

15 m

20°

x

28.

x

5°

2 m

29.

x

25°

√11 yd

30.

x / 63°   3√2 yd

Calculate the measure of angle *X* for each triangle. Round your answers to the nearest hundredth.

31.   *D*

10 m

16 m

V

X
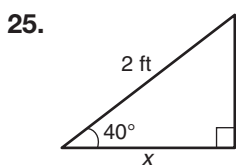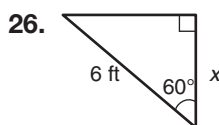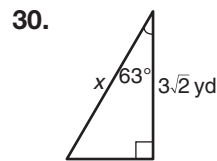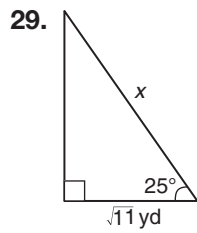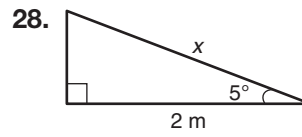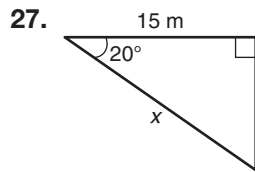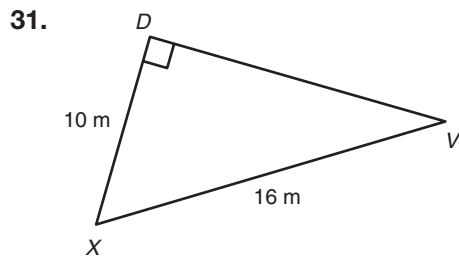
$$\cos X = \frac{10}{16}$$

$$m\angle X = \cos^{-1}\left(\frac{10}{16}\right) \approx 51.32°$$

32.   13 in.   *V*

X

9 in.

D

Name _____ Date _____

33.

34.

35.

36.
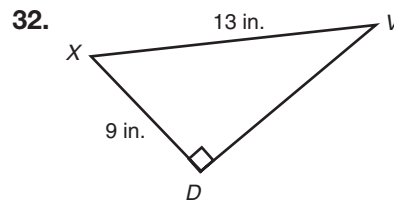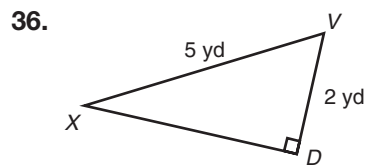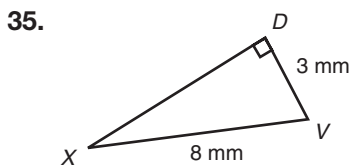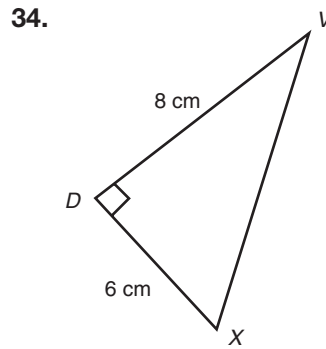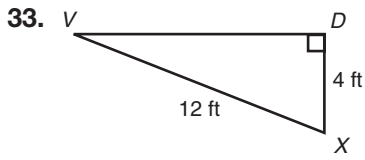
Solve each problem. Round your answers to the nearest hundredth.

37. The path of a model rocket is shown below. How far east did the rocket travel?
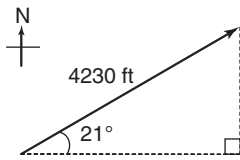
$$\cos 21° = \frac{E}{4230}$$

$$4230 \cos 21° = E$$

$$E \approx 3949.05 \text{ ft}$$

38. An ichthyologist tags a shark and charts its path. Examine his chart below. How far south did the shark travel?
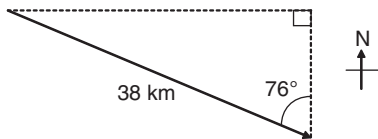
39. A kite is flying 120 feet away from the base of its string, as shown below. How much string is let out?
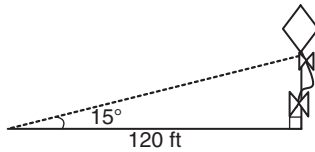
40. A pole has a rope tied to its top and to a stake 15 feet from the base. What is the length of the rope?
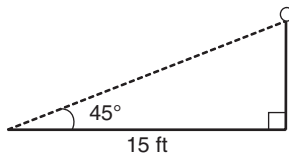
Name _____ Date _____

41. You park your boat at the end of a 20-foot dock. You tie the boat to the opposite end of the dock with a 35-foot rope. The boat drifts downstream until the rope is extended as far as it will go, as shown in the diagram. What is the angle formed by the rope and the dock?

35 ft

downstream

20 ft ?

Dock
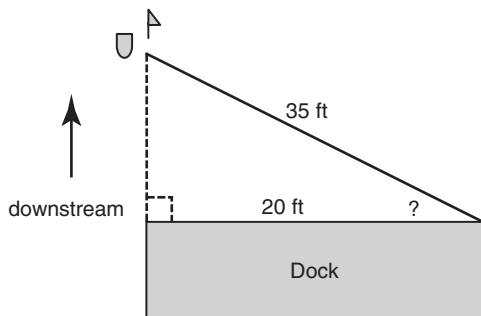
42. Rennie is walking her dog. The dog's leash is 12 feet long and is attached to the dog 10 feet horizontally from Rennie's hand, as shown in the diagram. What is the angle formed by the leash and the horizontal at the dog's collar?

Leash

12 ft

?

10 ft
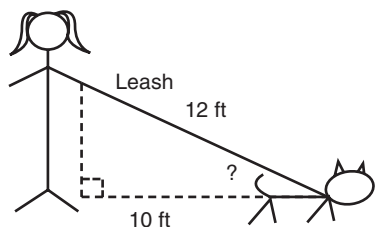
43. A ladder is leaning against the side of a house, as shown in the diagram. The ladder is 24 feet long and makes a 76° angle with the ground. How far from the edge of the house is the base of the ladder?
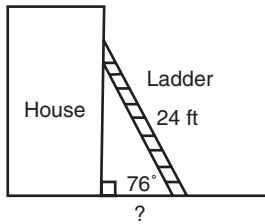
44. A rectangular garden 9 yards long has a diagonal path going through it, as shown in the diagram. The path makes a 34° angle with the longer side of the garden. Determine the length of the path.
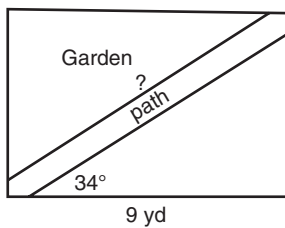
Name _____ Date _____

We Complement Each Other!
Complement Angle Relationships

Problem Set

For each right triangle, name the given ratio in two different ways.

1.
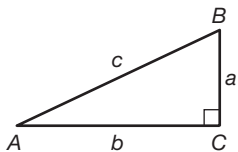
$\dfrac{a}{c}$

$\sin \angle A = \dfrac{a}{c}$

$\cos \angle B = \dfrac{a}{c}$

2.
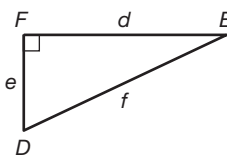
$\dfrac{d}{e}$

3.
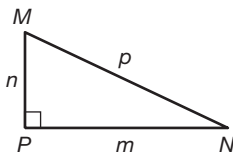
$\dfrac{p}{m}$

4.
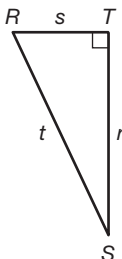
$\dfrac{s}{r}$

5.
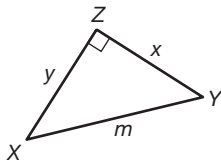
$\dfrac{y}{z}$

6.
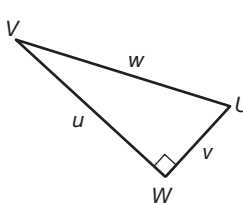
$\dfrac{w}{v}$

Determine the trigonometric ratio that you would use to solve for *x* in each triangle. Explain your reasoning. You do not need to solve for *x*.

7.

8 cm

35°

x
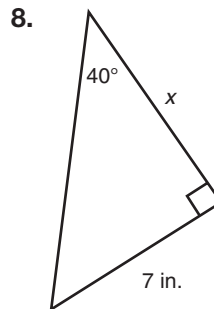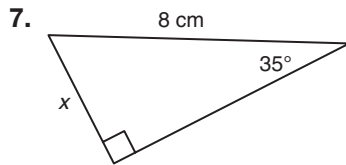
I would use the sine ratio because the hypotenuse is given and the length of the side opposite the given angle needs to be determined.

8.

40°

x

7 in.

9.

x

45°

$3\sqrt{2}$ m

10.

17 yd

60°

x

11.

x

3.1 mm

75°

12.

21 ft

17°

x
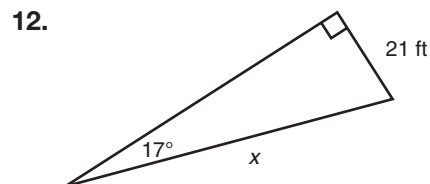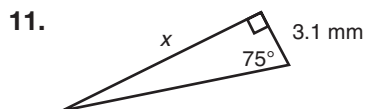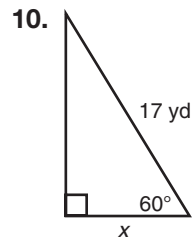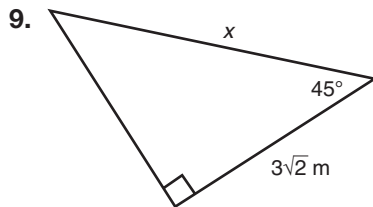
Name _____ Date _____

Solve each problem. Round your answers to the nearest hundredth.

13. You are standing 40 feet away from a building. The angle of elevation from the ground to the top of the building is 57°. What is the height of the building?

$$\tan 57° = \frac{h}{40}$$

$$40 \tan 57° = h$$

$$h \approx 61.59 \text{ ft}$$

14. A surveyor is 3 miles from a mountain. The angle of elevation from the ground to the top of the mountain is 15°. What is the height of the mountain?

15. The angle of elevation from a ship to a 135-foot-tall lighthouse is 2°. How far is the ship from the lighthouse?

16. The Statue of Liberty is about 151 feet tall. If the angle of elevation from a tree in Liberty State Park to the statue's top is 1.5°, how far is the tree from the statue?

17. The angle of elevation from the top of a person's shadow on the ground to the top of the person
is 45°. The top of the shadow is 50 inches away from the person. How tall is the person?

18. A plane is spotted above a hill that is 12,000 feet away. The angle of elevation to the plane is 28°.
How high is the plane?

19. During the construction of a house, a 6-foot-long board is used to support a wall. The board has an
angle of elevation from the ground to the wall of 67°. How far is the base of the wall from the board?

20. Museums use metal rods to position the bones of dinosaurs. If an angled rod needs to be placed
1.3 meters away from a bone, with an angle of elevation from the ground of 51°, what must the length
of the rod be?

Name _____ Date _____

Solve each problem. Round your answers to the nearest hundredth.

21. The angle of depression from the top of a building to a telephone line is 34°. If the building is 25 feet tall, how far from the building does the telephone line reach the ground?

$$\tan 34° = \frac{25}{d}$$

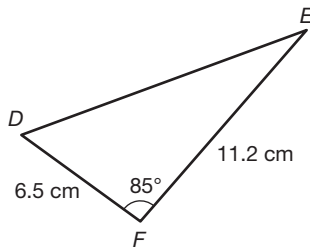$$d = \frac{25}{\tan 34°}$$

$$d \approx 37.06 \text{ ft}$$

22. An airplane flying 3500 feet from the ground sees an airport at an angle of depression of 77°. How far is the airplane from the airport?

23. To determine the depth of a well's water, a hydrologist measures the diameter of the well to be 3 feet. She then uses a flashlight to point down to the water on the other side of the well. The flashlight makes an angle of depression of 79°. What is the depth of the well water?
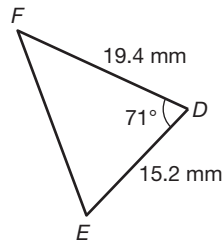
24. A zip wire from a tree to the ground has an angle of depression of 18°. If the zip wire ends 250 feet from the base of the tree, how far up the tree does the zip wire start?

25. From a 50-foot-tall lookout tower, a park ranger sees a fire at an angle of depression of 1.6°. How far is the fire from the tower?

26. The Empire State Building is 448 meters tall. The angle of depression from the top of the Empire State Building to the base of the UN building is 74°. How far is the UN building from the Empire State Building?

27. A factory conveyor has an angle of depression of 18° and drops 10 feet. How long is the conveyor?

28. A bicycle race organizer needs to put up barriers along a hill. The hill is 300 feet tall and from the top makes an angle of depression of 26°. How long does the barrier need to be?
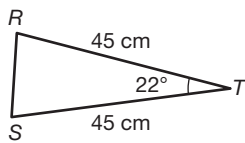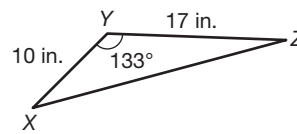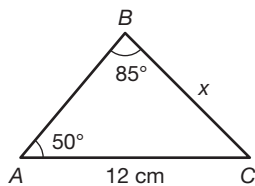
Name _____ Date _____

Time to Derive!
Deriving the Triangle Area Formula, the Law of Sines, and the Law of Cosines

Vocabulary

Define each term in your own words.

1. Law of Sines

2. Law of Cosines

Problem Set

Determine the area of each triangle. Round your answers to the nearest tenth.

1.

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2} (19)(16)(\sin 67°)$

$A \approx 139.9$

The area of the triangle is approximately 139.9 square centimeters.

2.

3.

4. F

5. R

6.

Determine the unknown side length *x* by using the Law of Sines. Round your answers to the nearest tenth.

7.

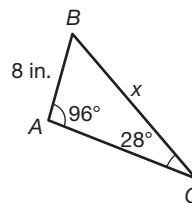$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50°}{x} = \frac{\sin 85°}{12}$$

$$12 \sin 50° = x \sin 85°$$
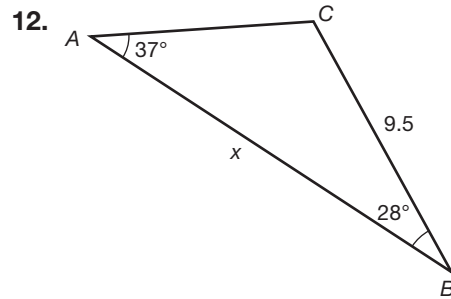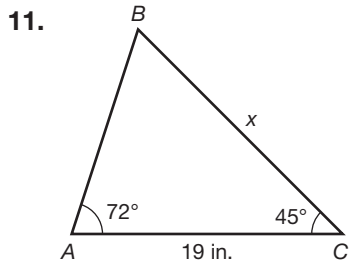
$$x = \frac{12 \sin 50°}{\sin 85°}$$

$$x \approx 9.2 \text{ cm}$$

8.

Name _____ Date _____

9.

10.

11.

12.
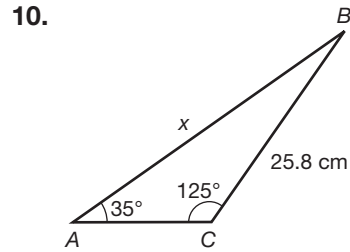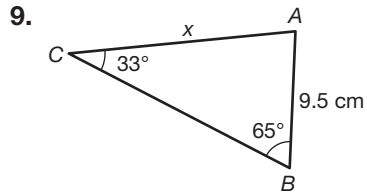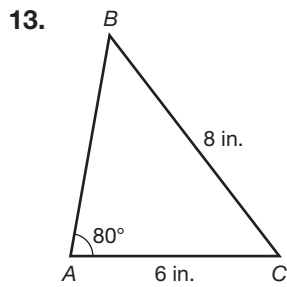
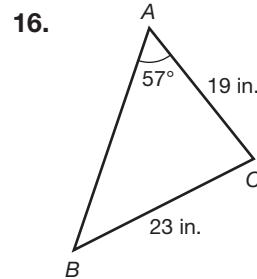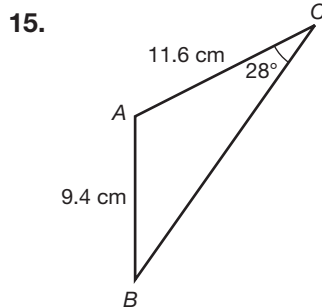Determine $m\angle B$ by using the Law of Sines. Round your answers to the nearest tenth.

13.
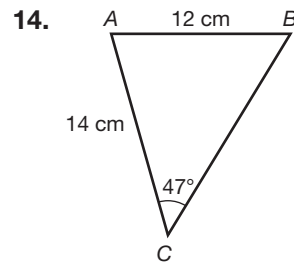

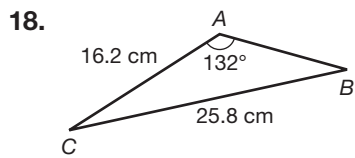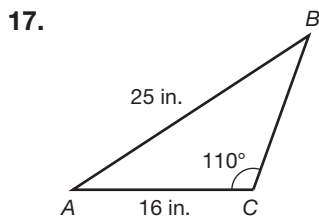$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{6} = \frac{\sin 80°}{8°}$$

$$8 \sin B = 6 \sin 80°$$
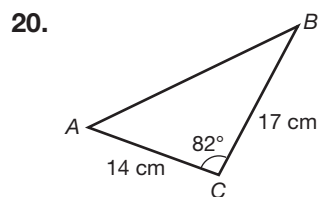
$$\sin B = \frac{6 \sin 80°}{8} \approx 0.739$$

$$m\angle B = \sin^{-1}(0.739) \approx 47.6°$$

14.

15.

16.

Name _____ Date _____

17.

25 in.

B

110°

A   16 in.   C

18.

A

16.2 cm   132°

B

25.8 cm

C

Determine the unknown side length by using the Law of Cosines. Round your answers to the nearest tenth.

19.

7 in.

42°
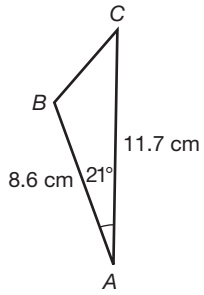
5 in.

20.

B

A

82°   17 cm

14 cm

C
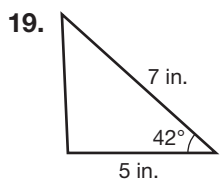
$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 5^2 + 7^2 - 2(5)(7)\cos 42°$

$b^2 = 25 + 49 - 70 \cos 42° \approx 21.98$

$b = \sqrt{21.98}$

$b \approx 4.7$ in.
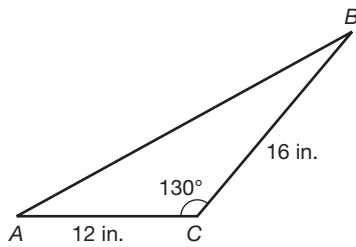
21.

22.

23.

24.